

TEST DE EVALUARE
ECUAȚII CU MODUL ȘI CU PARTE ÎNTREAGĂ
INECUAȚII CU MODUL

SUBIECTE

1. Aflați x real pentru care există inegalitățile:

a) $|-2x+3| \geq 7$ **(5 p)**

b) $|-2x-5| \leq |2x+1|$ **(10 p)**

2. Rezolvați în \mathbf{R} ecuațiile:

a) $[2x-1]=7$ **(5 p)**

b) $[3x-5]=x+18$ **(10 p)**

c) $\left[\frac{2x+13}{5}\right] = 3x - 11$ **(15 p)**

3. Rezolvați în \mathbf{R} ecuațiile:

a) $|x-3,25 + \frac{1}{4}| + |6-2x| + |5x+3 - (\frac{1}{18})^{-1}| = 72.$ **(10 p)**

b) $||2x-3| - 3| = 5$ **(10 p)**

c) $|2x+7| = x + 12$ **(15 p)**

d) $|7x-5| = |-2x+4|$ **(10 p)**

REZOLVAREA SUBIECTELOR

1.	<p style="text-align: center;">a) $-2x+3 \geq 7 \Rightarrow 2x - 3 \leq -7$ sau $2x - 3 \geq 7$ Din $2x - 3 \leq -7 \Rightarrow 2x \leq -4 \Rightarrow x \leq -2 \Rightarrow x \in (-\infty; -2]$ Din $2x - 3 \geq 7 \Rightarrow 2x \geq 10 \Rightarrow x \geq 5 \Rightarrow x \in [5; +\infty)$</p>
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	<p>Deci $x \in (-\infty; -2] \cup [5; +\infty) \Rightarrow S = (-\infty; -2] \cup [5; +\infty)$.</p>
	<p>b) $-2x-5 \leq 2x+1 \Rightarrow (-2x-5)^2 \leq (2x+1)^2$ $\Rightarrow 4x^2 + 20x + 25 \leq 4x^2 + 4x + 1 \Rightarrow 16x \leq -24 \Rightarrow x \leq -\frac{3}{2} \Rightarrow x \in (-\infty; -\frac{3}{2}]$.</p>
<p>2.</p>	<p>a) $[2x-1] = 7 \in \mathbf{Z} \Rightarrow 7 \leq 2x-1 < 8 \Rightarrow 8 \leq 2x < 9 \Rightarrow 4 \leq x < \frac{9}{2}$ $\Rightarrow x \in [4; \frac{9}{2})$.</p>
	<p>b) $[3x-5] = x+18$ Cum $[3x-5] \in \mathbf{Z} \Rightarrow x+18 \in \mathbf{Z} \Rightarrow x \in \mathbf{Z} \Rightarrow 3x-5 \in \mathbf{Z} \Rightarrow [3x-5] = 3x-5$ Din $[3x-5] = x+18 \Rightarrow 3x-5 = x+18 \Rightarrow 2x = 23 \Rightarrow x = \frac{23}{2} \notin \mathbf{Z} \Rightarrow S = \emptyset$.</p>
	<p>c) $[\frac{2x+13}{5}] = 3x-11$ Din $[\frac{2x+13}{5}] = k, k \in \mathbf{Z} \Rightarrow 3x-11 = k \Rightarrow x = \frac{k+11}{3}$. Avem $[\frac{2x+13}{5}] = k, k \in \mathbf{Z} \Rightarrow k \leq \frac{2 \cdot \frac{k+11}{3} + 13}{5} < k+1 \Rightarrow k \leq \frac{2k+22+39}{15} < k+1 \Rightarrow$ $\Rightarrow k \leq \frac{2k+61}{15} < k+1 \Rightarrow 15k \leq 2k+61 < 15k+15 \Rightarrow \frac{46}{13} < k \leq \frac{61}{13}$. $\left. \begin{array}{l} \frac{46}{13} < k \leq \frac{61}{13} \\ k \in \mathbf{Z} \end{array} \right\} \Rightarrow k = 4 \Rightarrow x = 5 \Rightarrow S = \{5\}$.</p>

3.

$$\text{a) } \left| x - 3,25 + \frac{1}{4} \right| + |6 - 2x| + \left| 5x + 3 - \left(\frac{1}{18} \right)^{-1} \right| = 72 \Rightarrow$$

$$\Rightarrow |x - 3,25 + 0,25| + |6 - 2x| + |5x + 3 - 18| = 72$$

$$\Rightarrow |x - 3| + |-2(x - 3)| + |5(x - 3)| = 72 \Rightarrow |x - 3| + 2|x - 3| + 5|x - 3| = 72 \Rightarrow$$

$$\Rightarrow 8|x - 3| = 72 \Rightarrow |x - 3| = 9 \Rightarrow x - 3 = \pm 9 \Rightarrow x_1 = 12, x_2 = -6.$$

$$\Rightarrow S = \{-6; 12\}.$$

$$\text{b) } \left| |2x - 3| - 3 \right| = 5 \Rightarrow |2x - 3| - 3 = \pm 5$$

$$\text{Din } |2x - 3| - 3 = 5 \Rightarrow |2x - 3| = 8 \Rightarrow 2x - 3 = \pm 8 \Rightarrow x_1 = \frac{11}{2}, x_2 = -\frac{5}{2}.$$

$$\text{Din } |2x - 3| - 3 = -5 \Rightarrow |2x - 3| = -2 < 0 \Rightarrow x \in \emptyset.$$

$$\text{Deci } S = \left\{ -\frac{5}{2}, \frac{11}{2} \right\}.$$

$$\text{c) } |2x + 7| = x + 12$$

$$\text{C.E. : } x + 12 \geq 0 \Rightarrow x \geq -12 \Rightarrow x \in [-12, +\infty)$$

$$\text{Avem } 2x + 7 = \pm(x + 12)$$

$$\text{Din } 2x + 7 = x + 12 \Rightarrow x = 5 \in [-12, +\infty)$$

$$\text{Din } 2x + 7 = -x - 12 \Rightarrow 3x = -19 \Rightarrow x = -\frac{19}{3} \in [-12, +\infty)$$

$$\text{Deci } S = \left\{ -\frac{19}{3}; 5 \right\}.$$

<p>d) $7x-5 = -2x+4$</p> <p>Dacă $x = y$ atunci $x = y$ sau $x = -y$.</p> <p>Din $7x - 5 = -2x + 4 \Rightarrow 9x = 9 \Rightarrow x = 1$.</p> <p>Din $7x - 5 = 2x - 4 \Rightarrow 5x = 1 \Rightarrow x = \frac{1}{5}$.</p> <p>Deci $S = \left\{\frac{1}{5}; 1\right\}$.</p>

BAREM DE EVALUARE ȘI DE NOTARE

SUBIECTUL 1

a)	$ -2x+3 \geq 7 \Rightarrow 2x - 3 \leq -7$ sau $2x - 3 \geq 7$2p Din $2x - 3 \leq -7 \Rightarrow 2x \leq -4 \Rightarrow x \leq -2 \Rightarrow x \in (-\infty; -2]$1p Din $2x - 3 \geq 7 \Rightarrow 2x \geq 10 \Rightarrow x \geq 5 \Rightarrow x \in [5; +\infty)$1p Deci $x \in (-\infty; -2] \cup [5; +\infty) \Rightarrow S = (-\infty; -2] \cup [5; +\infty)$1p
b)	$ -2x-5 \leq 2x+1 \Rightarrow (-2x - 5)^2 \leq (2x + 1)^2$1p $\Rightarrow 4x^2 + 20x + 25 \leq 4x^2 + 4x + 1$4p $\Rightarrow 16x \leq -24$2p $\Rightarrow x \leq -\frac{3}{2} \Rightarrow x \in (-\infty; -\frac{3}{2}]$3p

SUBIECTUL 2

a)	$[2x-1] = 7 \in \mathbf{Z} \Rightarrow 7 \leq 2x - 1 < 8$2p $\Rightarrow 8 \leq 2x < 9 \Rightarrow 4 \leq x < \frac{9}{2}$2p $\Rightarrow x \in \left[4; \frac{9}{2}\right)$1p
b)	$[3x-5] = x + 18$ Cum $[3x-5] \in \mathbf{Z} \Rightarrow x + 18 \in \mathbf{Z} \Rightarrow x \in \mathbf{Z}$3p

	$\Rightarrow 3x - 5 \in \mathbf{Z} \Rightarrow [3x-5] = 3x - 5 \dots\dots\dots 2p$ Din $[3x-5] = x + 18 \Rightarrow 3x - 5 = x + 18 \dots\dots\dots 3p$ $\Rightarrow 2x = 23 \Rightarrow x = \frac{23}{2} \notin \mathbf{Z} \Rightarrow S = \emptyset \dots\dots\dots 2p$
c)	$\left[\frac{2x + 13}{5} \right] = 3x - 11$ Din $\left[\frac{2x+13}{5} \right] = k, k \in \mathbf{Z} \Rightarrow 3x - 11 = k \Rightarrow x = \frac{k+11}{3} \dots\dots\dots 3p$ Avem $\left[\frac{2x+13}{5} \right] = k, k \in \mathbf{Z} \Rightarrow k \leq \frac{2 \cdot \frac{k+11}{3} + 13}{5} < k + 1 \dots\dots\dots 2p$ $\Rightarrow k \leq \frac{2k+22+39}{15} < k + 1 \Rightarrow k \leq \frac{2k+61}{15} < k + 1 \dots\dots\dots 2p$ $\Rightarrow 15k \leq 2k + 61 < 15k + 15 \Rightarrow \frac{46}{13} < k \leq \frac{61}{13} \dots\dots\dots 4p$ $\left. \begin{array}{l} \frac{46}{13} < k \leq \frac{61}{13} \\ k \in \mathbf{Z} \end{array} \right \Rightarrow k = 4 \Rightarrow x = 5 \Rightarrow S = \{5\} \dots\dots\dots 4p$

SUBIECTUL 3

a)	$\left x - 3,25 + \frac{1}{4} \right + 6-2x + \left 5x+3 - \left(\frac{1}{18} \right)^{-1} \right = 72 \Rightarrow$ $\Rightarrow x-3,25 + 0,25 + 6-2x + 5x+3 - 18 = 72 \dots\dots\dots 3p$ $\Rightarrow x-3 + 2 x-3 + 5 x-3 = 72 \dots\dots\dots 3p$ $\Rightarrow 8 x-3 = 72 \Rightarrow x-3 =9 \Rightarrow x - 3 = \pm 9 \dots\dots\dots 3p$ $\Rightarrow x_1 = 12, x_2 = -6. \Rightarrow S = \{-6; 12\} \dots\dots\dots 1p$
b)	$ 2x-3 - 3 = 5 \Rightarrow 2x-3 - 3 = \pm 5 \dots\dots\dots 2p$ Din $ 2x-3 - 3 = 5 \Rightarrow 2x-3 = 8 \Rightarrow 2x - 3 = \pm 8 \Rightarrow x_1 = \frac{11}{2}, x_2 = -\frac{5}{2} \dots\dots 4p$ Din $ 2x-3 - 3 = -5 \Rightarrow 2x-3 = -2 < 0 \Rightarrow x \in \emptyset \dots\dots\dots 3p$ Deci $S = \left\{ -\frac{5}{2}, \frac{11}{2} \right\} \dots\dots\dots 1p$
c)	$ 2x+7 = x + 12$ C.E. : $x + 12 \geq 0 \Rightarrow x \geq -12 \Rightarrow x \in [-12, +\infty) \dots\dots\dots 3p$ Avem $2x + 7 = \pm(x + 12) \dots\dots\dots 3p$

	<p>Din $2x + 7 = x + 12 \Rightarrow x = 5 \in [-12, +\infty)$.....4p</p> <p>Din $2x + 7 = -x - 12 \Rightarrow 3x = -19 \Rightarrow x = -\frac{19}{3} \in [-12, +\infty)$.....4p</p> <p>Deci $S = \left\{-\frac{19}{3}; 5\right\}$.....1p</p>
d)	<p>$7x-5 = -2x+4$</p> <p>Dacă $a = b , a, b \in \mathbf{R}$ atunci $a = b$ sau $a = -b$.....1p</p> <p>Din $7x - 5 = -2x + 4 \Rightarrow 9x = 9 \Rightarrow x = 1$4p</p> <p>Din $7x - 5 = 2x - 4 \Rightarrow 5x = 1 \Rightarrow x = \frac{1}{5}$4p</p> <p>Deci $S = \left\{\frac{1}{5}; 1\right\}$.....1p</p>